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Inertial effects on neutrino oscillations

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Abstract. The inertial effects on neutrino oscillations induced by the acceleration and angular velocity of a reference frame are calculated. Such effects have been analyzed in the framework of the solar and atmospheric neutrino problem.

1 Introduction

The long-standing problem of solar neutrino deficiency, i.e., the discrepancy between the measured ν_e flux predicted by various solar models [1,2] and the atmospheric neutrino problem [3], might be explained by the invocation of oscillations between the various flavors or generations of neutrinos. It is well known, in fact, that neutrino oscillations [4] can occur in the vacuum if the eigenvalues of the mass matrix are not all degenerate and the corresponding mass eigenstates are different from weak interaction eigenstates ν_e, ν_μ, ν_τ. The most often discussed version of this type of solution is the Mikheyev–Smirnov–Wolfeinstein (MSW) effect [5], in which the solar electron neutrinos can be almost completely converted into muon or tau neutrinos, because of the presence of matter in the Sun. Recently a quantum field theory of neutrino oscillations has been proposed by Blasone, Vitiello [6], and Sassaroli [7].

An alternative mechanism of neutrino oscillations, which does not require the neutrino to have a nonzero mass, was first suggested by Gasperini [8] and by Halprin and Leung [9] as a means to test the equivalence principle. In this mechanism, neutrino oscillations occur as a consequence of an assumed flavor-nondiagonal coupling of neutrinos to gravity that violates the equivalence principle. This line of research has been followed also in [10]. A new solution of the solar neutrino problem proposed in [11] uses the mechanism introduced by Ellis, Hagelin, Nanopoulos, and Srednicki [12], who investigate the effect on neutrino oscillation of quantum mechanics violation due to quantum gravity on neutrino oscillation.

The effect of gravitationally induced quantum mechanical phases in neutrino oscillation has been discussed in [13]. Ahluwalia and Burgard consider the gravitational effect on neutrino oscillations, showing that an external weak gravitational field of a star adds a new contribution to the phase difference. They also suggest that the new oscillation phase may have a significant effect on supernova explosions, since the extremely large fluxes of neutrinos are produced with different energies corresponding to the flavor states. This result has been also discussed by Bhattachya, Habib, and Mottola [14]. In their approach, they found that the possible gravitational effect appears at a higher order than that calculated in [13], having a magnitude of the order 10^{-9}, which is completely negligible in typical astrophysical applications.

Neutrino oscillations in curved space-time have also been studied by Piniz, Roy, and Wudka [15], who observe that spin flavor resonant transitions of neutrinos may occur in the vicinity of active galactic nuclei because of gravitational effects and the presence of a large magnetic field, and by Cardall and Fuller [16] who introduce an approach which shows that gravitational (e.g., Schwarzschild field) effects on neutrino oscillations are intimately related to the redshift.

The purpose of this paper is to calculate the contribution to neutrino oscillations induced by inertial effects arising from the acceleration and rotation of reference frames. As well known, these effects are relevant in interferometry experiments. In fact, by using an accelerated neutron interferometer, Bonse and Wroblewski were able to find the predicted phase shift [17]. Because of the validity of the equivalence principle, one expects that this effect occurs also in a gravitational field, as is verified by Colella, Overhausen, and Werner [18]. In addition, Mashhoon has derived a coupling of neutron spin to the rotation of a noninertial reference frame [19] from an extension of the hypothesis of locality; Atwood, et al. found the neutron Sagnac effect using an angular velocity of about 30 times that of Earth [20]; and finally, Papini, Cai, and Lloyd calculate the spin-rotation and spin-acceleration contributions to the helicity precession of fermions [21].

At present, there is strong evidence in favor of oscillations of solar and atmospheric neutrinos and of their nonzero masses. Such results have been found in different experiments: (1) solar neutrino experiments [22–26],

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(2) atmospheric neutrino experiments [27–31], and (3) the accelerator LSND experiment [32]. Nevertheless, we have to note that many other neutrino oscillation experiments with neutrinos produced by reactors and accelerators have not found any evidence of neutrino oscillations.

Recent reports indicate that the best fits in favor of neutrino oscillations are obtained for the following cases [33]:
(MSW) small-angle mixing region,

$$|m_2^2-m_1^2| \simeq (3\text{–}10)\times 10^{-6}\ \mathrm{eV}^2\,,\ \sin^2 2\theta \simeq (0.6\text{–}1.3)\times 10^{-2}\,;$$

(MSW) large-angle mixing region

$$|m_2^2 - m_1^2| \simeq (1\text{–}20)\times 10^{-5}\ \mathrm{eV}^2\,,\quad \sin^2 2\theta \simeq 0.5\text{–}0.9\,;$$

solar-vacuum oscillation

$$|m_2^2 - m_1^2| \simeq (0.5\text{–}1.1)\times 10^{-10}\ \mathrm{eV}^2\,,\quad \sin^2 2\theta \simeq 0.67/1\,;$$

atmospheric neutrino oscillation (see also [34,35])

$$|m_2^2 - m_1^2| \simeq (10^{-3}\text{–}10^{-2})\ \mathrm{eV}^2\,,\quad \sin^2 2\theta \geq 0.8\,,$$

$$|m_2^2 - m_1^2| \simeq (0.5\text{–}6)\times 10^{-3}\ \mathrm{eV}^2\,,\quad \sin^2 2\theta \geq 0.82\,;$$

LSND experiment

$$|m_2^2 - m_1^2| \simeq (0.2\text{–}10)\ \mathrm{eV}^2\,,\quad \sin^2 2\theta \simeq (0.2\text{–}3)\times 10^{-2}\,.$$

The term $|m_2^2 - m_1^2|$ is the mass-squared difference of neutrinos, and θ is the mixing angle. In the following we will consider only the case of solar and atmospheric neutrino oscillations in the vacuum.

The layout of the paper is the following. In Sect. 2, we discuss the Dirac equation in curved space-time, and we calculate the probability that neutrino flavor oscillations occur with respect to an accelerating and rotating observer. In Sect. 3, we discuss the phenomenological consequences of inertial effects on the solar and atmospheric neutrino problem. Conclusions are drawn in Sect. 4.

2 Neutrino oscillations induced by accelerations and rotations

As in [16], the generalized neutrino phase is given by

$$|\psi_f(\lambda)\rangle = \sum_j U_{fj} \mathrm{e}^{\mathrm{i}\int_{\lambda_0}^{\lambda} P\cdot p_{\mathrm{null}}\mathrm{d}\lambda'} |\nu_j\rangle\,, \tag{1}$$

where f is the flavor index and j the mass one. U_{fj} are the matrix elements transforming flavor and mass bases, P is the four-momentum operator generating space-time translation of the eigenstates, and $p^\mu_{\mathrm{null}} = \mathrm{d}x^\mu/\mathrm{d}\lambda$ is the tangent vector to the neutrino worldline x^μ, parameterized by λ. The covariant Dirac equation in curved space-time [36] is $[\mathrm{i}\gamma^\mu(x)D_\mu - mc/\hbar]\psi = 0$; the matrices $\gamma^\mu(x)$ are related to the usual Dirac matrices $\gamma^{\hat{a}}$ by means of the vierbein fields $e^{\hat{a}}_\mu(x)$, in which the Greek (Latin with hat) indices refer to curved (flat) space-time. D_μ is defined as $D_\mu = \nabla_\mu + \Gamma_\mu(x)$, in which ∇_μ is the usual covariant derivative and $\Gamma_\mu(x)$ is the spinorial connection defined by

$$\Gamma_\mu(x) = \frac{1}{8}[\gamma^{\hat{a}}, \gamma^{\hat{b}}] e^\nu_{\hat{a}} e_{\nu\hat{b};\mu}\,,$$

(the semicolon delimits the covariant derivative). The relations

$$\gamma^{\hat{a}}[\gamma^{\hat{b}}, \gamma^{\hat{c}}] = 2\eta^{\hat{a}\hat{b}}\gamma^{\hat{c}} - 2\eta^{\hat{a}\hat{c}}\gamma^{\hat{b}} - 2i\varepsilon^{\hat{d}\hat{a}\hat{b}\hat{c}}\gamma^5\gamma^{\hat{d}}\,,$$

where $\eta^{\hat{a}\hat{b}}$ is the metric tensor in flat space-time, $\varepsilon^{\hat{d}\hat{a}\hat{b}\hat{c}}$ is the totally antisymmetric tensor, $\gamma^5 = \mathrm{i}\gamma^{\hat{0}}\gamma^{\hat{1}}\gamma^{\hat{2}}\gamma^{\hat{3}}$, and $\{\gamma^5, \gamma^{\hat{a}}\} = 0$, allow us to recast the nonvanishing contribution from the spin connection in the form

$$\gamma^{\hat{a}} e^\mu_{\hat{a}} \Gamma_\mu = \gamma^{\hat{a}} e^\mu_{\hat{a}} \left\{ \mathrm{i}A_{G\mu}\left[-(-g)^{-1/2}\frac{\gamma^5}{2}\right]\right\}\,, \tag{2}$$

where

$$A^\mu_G = \frac{1}{4}\sqrt{-g}\, e^\mu_{\hat{a}} \varepsilon^{\hat{d}\hat{a}\hat{b}\hat{c}} (e_{\hat{b}\mu;\sigma} - e_{\hat{b}\sigma;\nu}) e^\nu_{\hat{c}} e^\sigma_{\hat{d}}\,, \tag{3}$$

and $g \equiv \det(g_{\mu\nu})$. $g_{\mu\nu}$ is the metric tensor of curved space-time. The momentum operator P_μ, used to calculate the phase of neutrino oscillations, is derived from the mass shell condition

$$(P_\mu + \hbar A_{G\mu}\gamma^5)(P^\mu + \hbar A^\mu_G \gamma^5) = -M_f^2 c^2\,, \tag{4}$$

where M_f^2 is the vacuum mass matrix in flavor base

$$M_f^2 = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger\,, \qquad U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\,. \tag{5}$$

θ is the vacuum mixing angle. Ignoring terms of the order $\mathcal{O}(\hbar^2 A_G^2)$ and $\mathcal{O}(\hbar A_G M_f)$, one finds that for relativistic neutrinos moving along generic trajectories parameterized by λ, the column vector of flavor amplitude

$$\chi(\lambda) = \begin{pmatrix} \langle \nu_e|\psi(\lambda)\rangle \\ \langle \nu_\mu|\psi(\lambda)\rangle \end{pmatrix} \tag{6}$$

satisfies the equation

$$\mathrm{i}\frac{\mathrm{d}\chi}{\mathrm{d}\lambda} = \left(\frac{M_f^2 c^2}{2} + \hbar p\cdot A_G \gamma^5 \right)\chi\,. \tag{7}$$

In deriving (7), one uses the relation $P^0 = p^0$ and $P^i \approx p^i$ [16]. In an accelerating and rotating frame, the vierbein fields $e^{\hat{a}}_\mu(x)$ are given by [37]

$$e^{\hat{0}}_0 = 1 + \frac{\vec{a}\cdot\vec{x}}{c^2}\,,\quad e^{\hat{0}}_m = 0\,,\quad e^{\hat{k}}_0 = \varepsilon^{\hat{k}\hat{l}\hat{m}}\omega^{\hat{l}}x^{\hat{m}}\,,\quad e^{\hat{k}}_l = \delta^k_l\,, \tag{8}$$

where $k, l, m = 1, 2, 3$, $x^\mu = (x^0, \vec{x})$ are the local coordinates for the observer at the origin, and $\vec{a}$, $\vec{\omega}$ are the acceleration and angular velocity of the frame, respectively. The components $e^\mu_{\hat{a}}(x)$ and $e_{\mu\hat{a}}(x)$ are calculated by the use of the metric tensors $g_{\mu\nu}$ and $\eta_{\hat{a}\hat{b}}$, with $g_{\mu\nu}$ determined by the element line [37]

$$\mathrm{d}s^2 = \left[\left(1 + \frac{\vec{a}\cdot\vec{x}}{c^2}\right)^2 + \left(\frac{\vec{\omega}\cdot\vec{x}}{c}\right)^2 - \frac{(\vec{\omega}\cdot\vec{\omega})(\vec{x}\cdot\vec{x})}{c^2}\right](\mathrm{d}x^0)^2$$
$$-2\mathrm{d}x^0\mathrm{d}\vec{x}\cdot\frac{(\vec{\omega}\wedge\vec{x})}{c} - \mathrm{d}\vec{x}\cdot\mathrm{d}\vec{x}\,. \tag{9}$$

Inserting (8) into (3), one gets the components of A^μ_G,

$$A^0_G = 0\,,\quad \vec{A}_G = \frac{\sqrt{-g}}{2}\frac{1}{1+\frac{\vec{a}\cdot\vec{x}}{c^2}}\left\{ 2\frac{\vec{\omega}}{c} - \frac{1}{c^2}[\vec{a}\wedge(\vec{x}\wedge\vec{\omega})]\right\}\,, \tag{10}$$

so that (7) becomes

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \mathcal{T}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix} \tag{11}$$

where $a_f \equiv \langle \nu_f|\psi(\lambda)\rangle, f = e, \mu$, and the matrix $\mathcal{T}$ is defined as

$$\mathcal{T} = \begin{bmatrix} -(\Delta/2)\cos 2\theta & (\Delta/2)\sin 2\theta - \hbar\vec{p}\cdot\vec{A}_G \\ (\Delta/2)\sin 2\theta - \hbar\vec{p}\cdot\vec{A}_G & (\Delta/2)\cos 2\theta \end{bmatrix} \tag{12}$$

up to the $(m_1^2 + m_2^2)c^2/2$ term, proportional to the identity matrix. Here $\Delta \equiv (m_2^2 - m_1^2)c^2/2$. We restrict this analysis to flavors e, μ, but the analysis works also for different neutrino flavors. To determine the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$, corresponding to a fixed value of the acceleration and angular velocity of the frame (i.e., for a fixed value of the affine parameter λ), one has to diagonalize the matrix $\mathcal{T}$. Using the standard procedure, one writes the mass eigenstates as a superposition of flavor eigenstates,

$$\begin{aligned} |\nu_1(\lambda)\rangle &= \cos\tilde{\theta}(\lambda)|\nu_e\rangle - \sin\tilde{\theta}(\lambda)|\nu_\mu\rangle\,, \\ |\nu_2(\lambda)\rangle &= \sin\tilde{\theta}(\lambda)|\nu_e\rangle + \cos\tilde{\theta}(\lambda)|\nu_\nu\rangle\,, \end{aligned} \tag{13}$$

where the mixing angle $\tilde{\theta}$ is defined in terms of the vacuum mixing angle

$$\tan 2\tilde{\theta} = \frac{\Delta\sin 2\theta - 2\hbar\vec{p}\cdot\vec{A}_G}{\Delta\cos 2\theta}\,. \tag{14}$$

We note that $\tilde{\theta} \to \theta$ as $\vec{A}_G \to 0$ (i.e., $\vec{a} \to 0, \vec{\omega} \to 0$). The corresponding eigenvalues are

$$\tau_{1,2} = \pm\sqrt{\frac{\Delta^2}{4}\cos^2 2\theta + \left[\frac{\Delta}{2}\sin 2\theta - (\vec{p}\cdot\vec{A}_G)\right]^2}\,. \tag{15}$$

We set $|\psi(\lambda)\rangle = a_1(\lambda)|\nu_1\rangle + a_2(\lambda)|\nu_2\rangle$, and (11) assumes the form

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\,, \tag{16}$$

where $a_i = \langle\nu_i|\psi(\lambda)\rangle, i = 1, 2$, and

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \tilde{U}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix}\,, \quad \tilde{U} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}\,. \tag{17}$$

We have used the condition $\mathrm{d}\tilde{\theta}/\mathrm{d}\lambda \approx 0$ in order that (16) is a diagonal matrix. This means that we neglect the variations of acceleration and angular velocity, with respect to the affine parameter λ, in comparing to their magnitudes. Equation (16) implies $a_i(\lambda) = a_i(0)\exp\alpha(\lambda)$, $\alpha(\lambda) \equiv \mathrm{i}\int_{\lambda_0}^{\lambda}\tau_i\mathrm{d}\lambda', i = 1, 2$. For the initial condition $|\psi(0)\rangle = |\nu_e\rangle$, the state $|\psi(\lambda)\rangle$ is

$$\begin{aligned} |\psi(\lambda)\rangle = {} & [\cos\theta_0\cos\tilde{\theta}\mathrm{e}^{\mathrm{i}\alpha} + \sin\theta_0\sin\tilde{\theta}\mathrm{e}^{-\mathrm{i}\alpha}]|\nu_e\rangle + \\ & [-\cos\theta_0\sin\tilde{\theta}\mathrm{e}^{\mathrm{i}\alpha} + \sin\theta_0\cos\tilde{\theta}\mathrm{e}^{-\mathrm{i}\alpha}]|\nu_\mu\rangle\,, \end{aligned} \tag{18}$$

where $\theta_0 = \tilde{\theta}(\lambda_0)$. The probability of observing an electronic neutrino is therefore

$$|\langle\nu_e|\psi(\lambda)\rangle|^2 = \cos^2(\theta_0 + \tilde{\theta})\sin^2\alpha + \cos^2(\theta_0 - \tilde{\theta})\cos^2\alpha\,. \tag{19}$$

Equation (19) shows that accelerating and rotating observers will experience a flavor oscillation of neutrinos. From the equivalence principle, one concludes that gravitational fields can induce neutrino oscillations; this is in agreement with [8]–[16]. It is interesting to discuss some particular case, e.g., frame acceleration or rotation, in order to estimate the contributions to neutrino oscillations when inertial effects are taken into account.

Table 1. Estimation of $|m_2^2 - m_1^2|$ as function of E_ν, $\sin 2\theta$ and fixed value of ω_c.

E_ν(MeV)	$\sin 2\theta$	$\|m_2^2 - m_1^2\|(\mathrm{eV}^2)$
1	1	10^{-13}
1	10^{-1}	10^{-12}
10	1	10^{-12}
10	10^{-1}	10^{-11}
50–60	1	10^{-10}

3 Inertial effects on solar and atmospheric neutrinos

Consequences on neutrino oscillations can be derived from (12) and (14). Let us suppose that the linear acceleration is zero, $\vec{a} = 0$, and the reference frame is rotating. In this situation, one can define a critical angular velocity ω_c such that the off-diagonal matrix elements of (12) vanish:

$$\Delta\sin 2\theta \approx \frac{2\hbar}{c}\,\vec{\omega}_c\cdot\vec{p}\,. \tag{20}$$

For ultrarelativistic neutrinos, $E_\nu \sim pc$, (20) reduces to

$$|m_2^2 - m_1^2| \approx \frac{4\hbar E_\nu\omega_c}{\sin 2\theta}\,. \tag{21}$$

This formula connects the mass-squared difference of neutrinos to the vacuum mixing angle, the neutrino energy, and the (critical) angular velocity of the reference frame. In order to determine the consequences of (21) for the solar neutrino problem (vacuum oscillations), we assume that the reference frame comoves with the Earth, i.e., its angular velocity is $\omega_c \sim 7\times 10^{-5}$ rad/sec. Results for typical values of the neutrino energies and vacuum mixing angle are reported in Table 1. The agreement with the experimental data comes from neutrinos with energy varying in the range 10–60 MeV. In this range, we find a mass-squared difference of the order 10^{-12}– 10^{-10} eV^2 for the vacuum mixing angle $10^{-1} \le \sin 2\theta \le 1$. In addition, we observe that an extreme value of $\tilde{\theta}$ as function of θ (14) is

$$\tilde{\theta} = \theta + \frac{\pi}{4}\,. \tag{22}$$

On the other hand, the adiabatic condition (20) implies $\tilde{\theta} \approx 0$ (which gives standard results) or $\tilde{\theta} \approx \pi/2$, which fixes the vacuum mixing angle to approximately to $\theta \approx \pi/4$, as is expected by experimental results for solar neutrinos.

The value $\tilde{\theta} \approx \pi/2$ induces a conversion phenomena for which the flux of the ν_e component decreases. To be more specific: after the production of electronic neutrinos in the Sun, we have $|\psi(0)\rangle = |\nu_e\rangle$ ($\theta_0 = 0$). Evolving along its worldline, the ν_e component will oscillate in agreement with (13). Nevertheless, if condition (14) holds and $\tilde{\theta} \approx \pi/2$, the probability (19) to find ν_e in the beam decreases from 1 to $\sin^2\theta_0 \approx 0$. This result shows that the ν_e component of the beam is almost totally depleted with respect to the rotating observer with angular velocity ω_c, resulting in a reduction of solar neutrino flux.

Concerning the atmospheric neutrino oscillation, an appreciable value of $|m_2^2 - m_1^2|$ requires highly (multiple GeV) energetic neutrinos. In fact, (21) implies $|m_2^2 - m_1^2| \sim 10^{-4}$ eV^2, for $E_\nu \sim 10^4$ GeV and $\sin 2\theta \sim 10^{-2}$. This value of the mixing angle is excluded (at least for now) by experimental data. When $\sin^2 2\theta \geq 0.82$ is used, according to the experimental results, (21) leads to a mass-squared difference of the order 10^{-6} eV^2, which does not fit the experimental range 10^{-4}–10^{-3} eV^2.

In the regime in which neutrinos are highly energetic, so that the condition $\vec{p}\cdot\vec{A}_G \gg \Delta \sin 2\theta$ holds, (14) implies $\tilde{\theta} \approx \pi/4$, and the probability to find the ν_e component in the neutrino beam is $\approx 1/2$, assuming as an initial condition $|\psi(0)\rangle = |\nu_e\rangle$, $\theta_0 = 0$. Highly energetic neutrinos, with energy of the order 1–10^3 TeV can be produced, for example, by a supernova [15].

It is interesting to compare the contribution to neutrino oscillations due to the rotation term with that due to the massive term. Being that $p \cdot A_G = -(\sqrt{-g}/c)\vec{\omega}\cdot\vec{p}$, (7) implies, for ultrarelativistic neutrinos,

$$\omega \sim \frac{1}{2}\frac{m_\nu^2 c^4}{\hbar E_\nu}\,. \tag{23}$$

If we consider neutrinos with energy $E_\nu \sim 1$ TeV emanating from active galactic nuclei [15], which are possible sources of high- energy signals being the most luminous objects in the universe [38], then for $m_\nu \sim 1$ eV/c^2 [15,40], the angular velocity is of the order $\omega \sim 10^2$–10^3 rad/sec. Moreover, one expects that the neutrino mass is of the order $m_\nu \sim 10^{-2}$–10^{-4} eV/c^2 [41],[42]. At energy $E_\nu \sim 10$–10^2 GeV produced by accelerators [39], one gets $\omega \sim 1.2$–10^{-5} rad/sec.

Some values of the angular velocity, calculated by using (23) for different (and expected) values of neutrino masses and energies, are reported in Table 2. It turns out that these values are of the same order of magnitude as the typical angular velocity of astrophysical objects (Table 3). For example, the angular velocity $\omega \sim 10^2$–10^3 rad/sec is comparable with that of pulsars [43]. In addition, $\omega \sim 10^{-5}$ rad/sec is of the same order of angular velocity of the Earth, and it is about 10 times the angular velocity of the Sun: $\omega_{Sun} \sim 10^{-6}$ rad/sec.

In the case in which the acceleration $\vec{a}$ is constant and $\vec{\omega} = 0$, the $p \cdot A_G$ term in (7) vanishes, and one finds a shift of the phase: $\Omega = \mathrm{i}\int_{\lambda_0}^{\lambda} P \cdot p_{\mathrm{null}}\mathrm{d}\lambda'$, as defined in (1). In fact, since the neutrino trajectory is null and $g_{\mu\nu}$ is diagonal ($\sqrt{-g} = 1 + (\vec{a}\cdot\vec{x})/c^2$), the physical distance can be written as

Table 2. Angular velocity of reference frames for different values of neutrino masses and energies.

m_ν(eV/c^2)	E_ν(GeV)	ω(rad/sec)
1	10^3–10^{-1}	10^2–10^6
10^{-2}	10^3–10^{-1}	10^{-2}–10^2
10^{-4}	10^3–10^{-1}	10^{-6}–10^{-2}

Table 3. Typical angular velocity of Astrophysical objects.

Astrophysical Objects	ω(rad/sec)[43]
Pulsar	4×10^3
Sun	10^{-6}
Earth	10^{-5}
White dwarf	2.1
RR Lyrae Star	10^{-5}
Cepheid Variable	10^{-7}

$$\mathrm{d}\lambda = \mathrm{d}l\left(g_{ij}\frac{\mathrm{d}x^i}{\mathrm{d}\lambda}\frac{\mathrm{d}x^j}{\mathrm{d}\lambda}\right)^{-1/2} = \mathrm{d}l\left[-g_{00}\left(\frac{\mathrm{d}x^0}{\mathrm{d}\lambda}\right)^2\right]^{-1/2}, \tag{24}$$

and one gets [16]

$$\Omega = -\frac{M^2}{2E_*}\int_{l_0}^{l}\frac{1}{1+(\vec{a}\cdot\vec{x})/c^2}\mathrm{d}l'\,, \tag{25}$$

where $E_* = P_t$ is the conserved quantity due to the non dependence of the metric tensor on the timelike coordinates. If $\vec{a} \parallel \vec{x}$, then $\mathrm{d}l = \mathrm{d}r$, and (25) reduces to

$$\Omega = -\frac{M^2c^2}{2E_*|\vec{a}|}\ln(1+|\vec{a}|r/c^2)\,,$$

for $r_0 \equiv r(l_0 = 0) = 0$. If $\vec{a} \perp \vec{x}$, (25) gives the standard result $\Omega = -(M^2c^2/(2E_*)(\lambda - \lambda_0)$.

4 Conclusions

We have analyzed the phenomenological aspects of neutrino oscillations for an accelerating and rotating observer.

The inertial effects on neutrino oscillations seem to be appreciable in the solar neutrino problem. The coupling of the angular velocity and momentum of the neutrino implies a reduction of neutrino flux as experienced by an observer comoving with the Earth, providing us with valuable information on the mass-squared difference and mixing angle of neutrinos. In fact, we find good agreement between the experimental data [34] and our estimations of the mass-squared differences. They fit well the experimental data for neutrino beams with energies of the order 10–60 MeV. For these estimations, we have used $\sin 2\theta \sim 1$–10^{-1}, values coming from the data of solar neutrino oscillation experiments.

In the framework of atmospheric neutrinos, inertial effects seem to be negligible. The best fit of experimental data that we reproduce, if the adiabatic condition holds, is for highly energetic neutrinos. In fact, we get a value of the mass-squared difference of the order 10^{-4} eV^2, requiring a mixing angle $\sin 2\theta \sim 10^{-2}$.

However, we note that values of the mass-squared difference of neutrinos and their mixing angle have been until now open issues. Only future neutrino oscillation experiments will make it possible to investigate in detail the region $|m_2^2 - m_1^2|$ and to fix the value of the mixing angle. These data will allow the definitive solution the solar and atmospheric neutrino problems and an understanding of whether inertial effects are important for explaining the deficit of solar and atmospheric neutrino flux.

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